Then determine the points R', R'' ... $R^{(m)}$ in which PQ cuts the first, and the corresponding pairs of points S_1', S_2', S_1'', S_2'' .... $S_1^{(m)}$, $S_2^{(m)}$ in which it cuts the second, and draw $R'M_1' = \sqrt{-1} \cdot PS_1'$, $R'M_2' = \sqrt{-1} \cdot PS_2'$, ... $R^{(m)} M_1^{(m)} = \sqrt{-1} \cdot PS_1^{(m)}$, $R^{(m)} M_2^{(m)} = \sqrt{-1} \cdot PS_2^{(m)}$.

The loci of M, R, S are the principal scalar and clinant radical loci of the formation $f(x, y)$.

From this it is concluded, that if $O_1 O_2 \ldots O_n O_1$ be a completely enclosed polygon, the sides of which are taken in order, to replace PQ, first in the direction $O_1 O_2$, $O_2 O_3$, &c., and then in the direction $O_2 O_1$, $O_3 O_2$, &c., and the points M_1, $M_2 \ldots M^n$, M_1', $M'_2 \ldots M_1^{(m)}$, $M_2^{(m)}$ be determined for each direction and each side, and $[O_{1,2} M \div O_{2,1} M]$ represent the product of the ratios $O_1 M \div O_2 M$, &c., the $O_1 M$ referring to the origin O_1 and the M being determined with reference to the direction $O_1 O_2$, and the $O_2 M$ referring to the origin O_2 and the M being determined with reference to the direction $O_2 O_1$, and so on, then we shall have in all cases,

$$[O_{1,2} M \div O_{2,1} M] \cdot [O_{2,3} M \div O_{3,2} M] \ldots . [O_{n,1} M \div O_{1,n} M] = 1,$$

and not $= \pm 1$, as supposed by Plücker, whose error is traced to its origin, and displayed in all the examples he has given, where the roots of $f = 0$ are scalar. This result is then applied to a simple case where some of the roots of $f = 0$ are clinant, and a result obtained in accordance with other considerations.

II. "A Seventh Memoir on Quantics." By Arthur Cayley, Esq., F.R.S. Received February 28, 1861.

(Abstract.)

The present memoir relates chiefly to the theory of ternary cubics. Since the date of my Third Memoir on Quantics, M. Aronhold has published the continuation of his researches on ternary cubics, in the memoir "Theorie der homogenen Functionen dritten Grades von drei Veränderlichen," Crelle, t. lv. pp. 97–191 (1858). He there considers two derived contravariants, linear functions of the fundamental ones, and which occupy therein the position which the funda-

mental contravariants PU, QU do in my third memoir; in the notation of the present memoir these derived contravariants are

$$\begin{aligned} \mathrm{YU} &= 3\mathrm{T}\ .\qquad \mathrm{PU} - 4\mathrm{S.QU},\\ \mathrm{ZU} &= -48\mathrm{S}^2\ .\ \mathrm{PU} + \mathrm{T}\ .\ \mathrm{QU}\ ; \end{aligned}$$

and for the canonical form $x^3+y^3+z^3+6lxyz$, they acquire respectively the factor $(1+8l^3)^2$, viz. in this case

$$\begin{aligned} \mathrm{YU} &= (1+8l^3)^2\left\{l\qquad\ (\xi^3+\eta^3+\zeta^3) - \quad 3\xi\eta\zeta\right\}\\ \mathrm{ZU} &= (1+8l^3)^2\left\{(1+2l^3)\,(\xi^3+\eta^3+\zeta^3) + 18l^2\xi\eta\zeta\right\}. \end{aligned}$$

The derived contravariants have with the covariants U, HU, even a more intimate connexion than have the contravariants PU, QU; and the advantage of the employment of YU, ZU fully appears by M. Aronhold's memoir.

But the conclusion is, not that the contravariants PU, QU are to be rejected, but that the system is to be completed by the addition thereto of two derived covariants, linear functions of U, HU; these derived covariants, suggested to me by M. Aronhold's memoir, are in the present memoir called CU, DU; their values are

$$\begin{aligned} \mathrm{CU} &= -\mathrm{T}\ .\ \mathrm{U} + 24\mathrm{S.HU}\\ \mathrm{DU} &= \ \ 8\mathrm{S}^2\ .\ \mathrm{U} - 3\mathrm{T}\ .\ \mathrm{HU}: \end{aligned}$$

and for the canonical form $x^3+y^3+z^3+6lxyz$, they acquire respectively, not indeed $(1+8l^3)^2$, but the simple power $(1+8l^3)$, as a factor, viz. in this case

$$\begin{aligned} \mathrm{CU} &= (1+8l^3)\left\{(-1+4l^3)\,(x^3+y^3+z^3) + \qquad 18lxyz\right\}\\ \mathrm{DU} &= (1+8l^3)\left\{l^2(5+4l^3)\,(x^3+y^3+z^3) + 3(1-10l^3)xyz\right\}; \end{aligned}$$

it was in fact by means of this condition as to the factor $(1+8l^3)$, that the foregoing expressions for CU, DU were obtained.

The formulæ of my third memoir and those of M. Aronhold are by this means brought into harmony and made parts of a whole; instead of the two intermediates $\alpha\mathrm{U}+6\beta\mathrm{HU}$, $6\alpha\mathrm{PU}+\beta\mathrm{QU}$ in Tables 68 and 69 of my Third Memoir, or of the intermediates $\alpha\mathrm{U}+6\beta\mathrm{HU}$ $-2\alpha\mathrm{YU}+2\beta\mathrm{ZU}$ of M. Aronhold's theory, we have the four intermediates $\alpha\mathrm{U}+6\beta\mathrm{HU}$, $-2\alpha\mathrm{YU}+2\beta\mathrm{ZU}$, $2\alpha\mathrm{CU}-2\beta\mathrm{DU}$, $6\alpha\mathrm{PU}+\beta\mathrm{QU}$ in Tables 74, 75, 76, and 77 of the present memoir. These four tables embrace the former results, and the new ones which relate to the covariants CU, DU; and they are what is most important in the pre-

sent memoir. I have, however, excluded from the Tables, and I do not in the memoir consider (otherwise than incidentally) the covariant of the sixth order ΘU, or the contravariant (reciprocant) FU.

I have given in the memoir a comparison of my notation with that of M. Aronhold. A short part of the memoir relates to the binary cubic and the binary quartic, viz. each of these quantics has a covariant of its own order, forming with it an intermediate $\alpha U+\beta W$, the covariants whereof contain quantics in (α, β), the coefficients of which are invariants of the original quantic. The formulæ which relate to these cases are in fact given in my Fifth Memoir, but they are reproduced here in order to show the relations between the quantics in (α, β) contained in the formulæ. As regards the binary quartic, these results are required for the discussion of the like question in regard to the ternary cubic, viz. that of finding the relations between the different quantics in (α, β) contained in the formulæ relating to the ternary cubic. Some of these relations have been obtained by M. Hermite in the memoir "Sur les formes cubiques à trois indéterminées," Liouville, t. iii. pp. 37–40 (1858), and in that "Sur la Résolution des équation du quatrième degré," Comptes Rendus, t. xlvi. p. 715 (1858), and by M. Aronhold in his memoir already referred to; and in particular I reproduce and demonstrate some of the results in the last mentioned memoir of M. Hermite. But the relations in question are in the present memoir exhibited in a more complete and systematic form.

III. "On the Secular Change in the Magnetic Dip in London, between the years 1821 and 1860." By Major-General Edward Sabine, R.A., Treas. and V.P.R.S. Received March 7, 1861.

I propose in this communication to bring together and discuss four determinations at different epochs, in which I have myself been either directly or indirectly concerned, which have had expressly in view the object which forms the title of the paper.

Epoch of 1821.—The experiments on this occasion were made in a part of the Regent's Park, then occupied as the nursery garden of Mr. Jenkins: an unexceptionable locality in all respects, and far